Enegies of harmonic oscillator

For the groundstate wavefunction:

$$\Psi_0 = N_0 e^{-\frac{\alpha x^2}{2}} \rightarrow E_0 = \frac{1}{2}\hbar\omega_0$$

$$\Psi_n = N_n H_n(\alpha^{\frac{1}{2}}x) e^{-\alpha\frac{x^2}{2}} \rightarrow E_n = ?$$

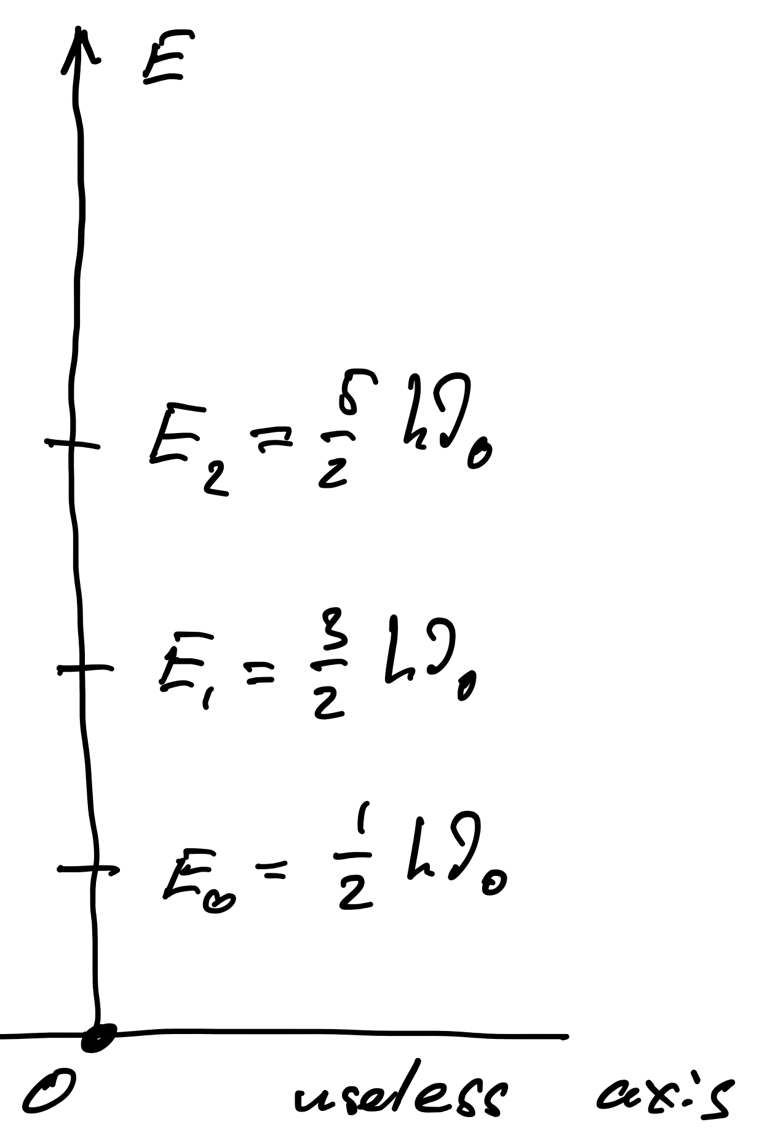
We will simply need to input it to Shrödinger equation:

$$\Psi_1 = N_1 x e^{-\frac{\alpha x^2}{2}} \rightarrow E_1 = \frac{3}{2}\hbar\omega_0$$

$$\Psi_2 = N_2(4\alpha x^2 - 2) \rightarrow E_2 = \frac{5}{2}\hbar\omega_0$$

$$\vdots \qquad\qquad \vdots$$

$$\Psi_n = N_n H_n(\alpha^{\frac{1}{2}}x) e^{-\frac{\alpha x^2}{2}} \rightarrow E_n = (n+\frac{1}{2})\hbar\omega$$

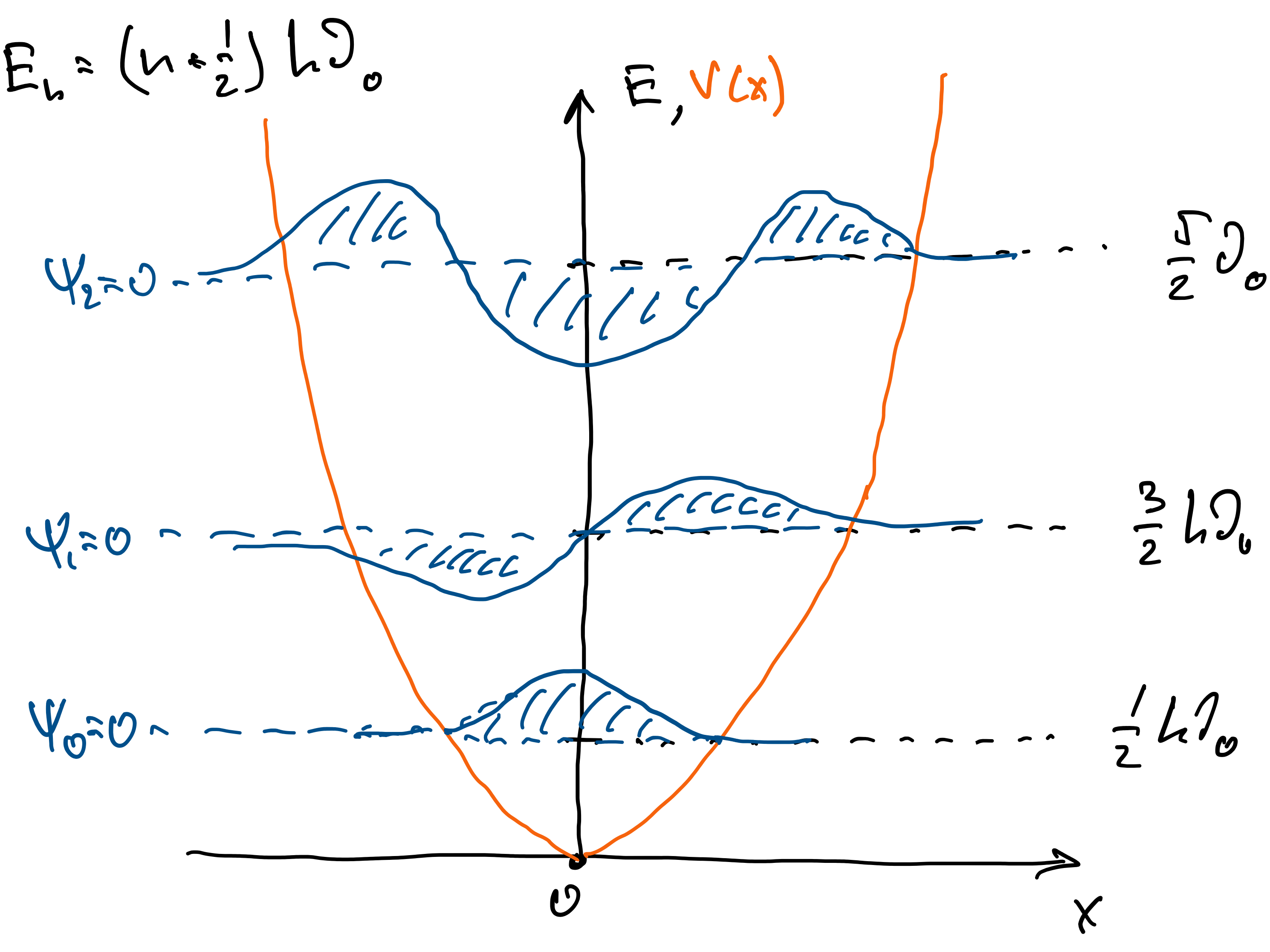


If we to summarize the picture for harmonic oscillator

$$V(x) = \frac{1}{2}kx^2$$

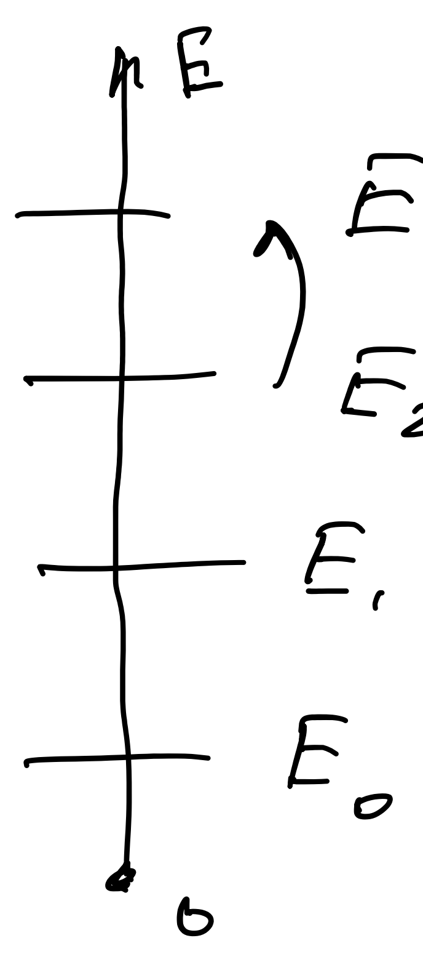
$$\Psi_n(x) = N_n H_n(\alpha^{\frac{1}{2}}x) e^{-\frac{\alpha x^2}{2}}$$

$$E_n = (n+\frac{1}{2})\hbar\omega_0$$



Let's see how selection rules work for HO

$$n \rightarrow n' \quad \int \Psi_n^* \mu_z \Psi_{n'} dx$$



The problem here is μ_z.

It is not a constant anymore,

as $\mu = \mu_0 + \frac{\partial\mu}{\partial x}x$

$$\int \Psi_n^* \mu_z \Psi_{n'} dx = \int \Psi_n^* (\mu_0 + \frac{\partial\mu}{\partial x}x) dx =$$

$$= \mu_0 \underbrace{\int \Psi_n^* \Psi_{n'} dx}_{0} + \frac{\partial\mu}{\partial x}\int \Psi_n^* x \Psi_{n'} dx$$

1) if $\frac{\partial\mu}{\partial x} = 0$, then transition is forbidden

obvious $\frac{\partial\mu}{\partial x} = 0 \rightarrow$ no oscillation

HCl vs H_2

2) $\int \Psi_n^* x \Psi_{n'} dx = 0$

$$\Psi_n = N_n H_n(\alpha^{\frac{1}{2}}x) e^{-\alpha\frac{x^2}{2}}$$

who cares?

$$\int N_n^* H_n x N_{n'} H_{n'} e^{-\alpha x^2} = N_n^* N_{n'} \int H_n x H_{n'} e^{-\alpha x^2} =$$

$$H_{n+1} = 2xH_n - 2nH_{n-1}$$

$$xH_n = \frac{1}{2}H_{n+1} - nH_{n-1}$$

$$= N_n^* N_{n'} \int H_n (\frac{1}{2}H_{n'+1} - n'H_{n'-1}) e^{-\alpha x^2} dx =$$

$$= A \underbrace{\int H_n H_{n'+1} dx}_{\substack{\neq 0 \\ n = n'+1}} - B \underbrace{\int H_n H_{n'-1} dx}_{\substack{\neq 0 \\ n = n'-1}}$$

$$\boxed{\Delta n = \pm 1} \; !$$